# **ACALANES UNION HIGH SCHOOL DISTRICT COURSE OF STUDY: CURRICULAR AREA - MATH**

COURSE TITLE: ALGEBRA 2

**GRADE LEVEL:** 9-12

**COURSE LENGTH:** One Year

PREFERRED PREVIOUS Algebra 1, Geometry

**COURSE OF STUDY:** 

**CREDIT:** 10 Credits

**UC/CSU CREDIT:** Meets UC/CSU credit for mathematics requirement; subject area ("c")

**GRADUATION** Fulfills 10 units of mathematics credit (2 semesters beyond Algebra 1) required for graduation

**REQUIREMENT:** 

**STANDARDS AND** 

California State Standards Algebra 2 **BENCHMARKS**:

ADOPTED: May 15, 2017

**INSTRUCTIONAL MATERIALS:** Big Ideas Learning: Big Ideas Algebra 2

## COURSE DESCRIPTION:

Algebra 2 complements and expands the mathematical concepts of Algebra 1 and Geometry. Topics include the real and complex number systems. Families of functions studied include: linear, quadratic, polynomial, exponential, logarithmic, rational and radical functions. Other units included are statistics, sequences and series, and the study of parabolas as conic sections. An introduction to trigonometry includes the unit circle and graphing trigonometric functions.

## **COURSE OBJECTIVES:**

## **Content Objectives**

During the course the students will:

- 1. Develop and extend an understanding of congruence using transformation, proving theorems and making constructions
- 2. Understand similarity using transformations and proving theorems
- 3. Define trigonometric ratios and solve problems involving triangles
- 4. Apply the definitions and properties of the parts of a circle to solving problems
- 5. Use coordinate geometry to prove geometric theorems
- 6. Visualize polygons and solids and calculate their perimeters, areas and volumes
- 7. Analyze real world applications of probability

# During the course students will work toward:

- 1. Active learning through investigation and conjecture
- 2. Consistent study habits, organization, and personal responsibility for learning
- 3. Proficiency in writing about mathematics and fluency in using mathematical vocabulary
- 4. Strategic implementation of instructional technologies
- 5. Effective communication and collaboration

#### ASSESSMENT:

Assessments are designed to promote and evaluate mathematical thinking. Teachers use engaging activities that involve students in investigating, conjecturing, verifying, applying, evaluating, and communicating in various assessment modalities.

Formal and informal assessments can be made on the basis of both individual and group work. Assessments should be from a variety of means and could include performance tasks, quizzes, tests, projects, investigations, and daily assignments.

Assessments should be measuring the following claims:

Claim #1 – Concepts & Procedures

Students can explain and apply mathematical concepts, as well as interpret and carry out mathematical procedures with precision and fluency.

Claim #2 - Problem Solving

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

Claim #3 – Communicating Reasoning

Students can clearly and precisely construct viable arguments to support their own reasoning and critique the reasoning of others.

Claim #4 - Modeling and Data Analysis

Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

### Performance Tasks

Performance tasks are used to better measure capacities such as depth of understanding, research skills, and complex analysis, which cannot be adequately assessed with selected- or constructed-response items. These tasks may require students to evaluate, optimize, design, plan, model, transform, generalize, justify, interpret, represent, estimate, and calculate solutions. Performance tasks can be used for a variety of purposes such as topic engagement, formative assessment and evaluative assessment. They may be performed individually or in small groups, depending on the purpose of the assessment.

## Performance tasks should:

- Integrate knowledge and skills across multiple claims
- Require student-initiated planning and management of information and ideas
- Reflect a real-world task and/or scenario-based problem
- Allow for multiple approaches
- Represent content that is relevant and meaningful to students
- Be assessed using an understandable rubric that provides meaningful feedback for students and the teacher

#### **GRADING GUIDELINES:**

See AUHSD Grading Guidelines: Final Mark Rubric and Final Course Mark Determination Components

### COURSE CONTENT:

## Mathematical Practices

The Standards for Mathematical Practice are "habits of the mind of mathematically proficient students". They describe the attributes that mathematics educators at all levels are striving to develop in their students, as these practices are based on key mathematical processes and proficiencies. The goal of implementing these practices is to develop students who can use their knowledge of mathematics in flexible, sophisticated, and relevant ways across multiple disciplines.

#1 Make sense of problems and persevere in solving them (Hypothesize & Strategize)

- Students are:
  - o Making conjectures about what the problem is asking and how they can begin to solve it
  - o Checking for the reasonableness of the strategy as the work progresses and making adjustments as needed
- Teachers develop this skill by having students:
  - o Explain the meaning of the problem and/or restate the problem
  - o Analyze the given information and develop entry points into the problem and develop strategies for solving the problem
  - Execute and evaluate multiple strategies

#2 Reason abstractly and quantitatively (De/Contextualize)

- Students are:
  - o Determining what numbers and symbols represent through the use of diagrams, graphs or equations
- Teachers develop this skill by having students:
  - Move between multiple representations to determine the meaning behind quantities
  - Express purely mathematical expressions with real world context and taking quantities out of context and representing them as abstract mathematical ideas or expressions

#3 Construct viable arguments; critique others' reasoning

- Students are:
  - Justifying their thinking by providing evidence based on mathematical properties and using that evidence to evaluate the reasoning of others
- Teachers develop this skill by having students:
  - Make conjectures, compare and contrast methods, and identify flawed logic by providing counter-example

#### #4 Model with Mathematics

- Students are:
  - o Interpreting and constructing graphs, tables, number lines, diagrams or equations to model real-world situational data
- Teachers develop this skill by having students:
  - o Use models to make interpolative and extrapolative inferences
  - Examine the effectiveness and appropriateness of a model

## #5 Use appropriate tools strategically

- Students are:
  - Selecting appropriate math tools and technology to help solve problems including manipulatives, graphing utilities, tables, [matrices], computer applications, compasses, etc.
- Teachers develop this skill by having students:
  - o Identify the strengths and weaknesses of different tools in relation to solving a given problem and also use tools to explore, confirm or deepen understanding

#### #6 Attend to Precision

- Students are:
  - o Calculating quantities accurately through proper rounding (based on context), labeling of units of measure, and checking their work
  - Selecting a problem solving method that allows for appropriate precision
- Teachers develop this skill by having students:
  - o Formulate precise explanations of their work using vocabulary and justify their rounding process
  - Re-examine their work or thinking process, and then demonstrate the method by which they check their answers

#### #7 Look For and Make Use of Structure

- Students are:
  - Looking for patterns or relationships and using that structure to simplify complex ideas
- Teachers develop this skill by having students:
  - Extend prior knowledge of similar situations to novel ones or break down complex problems in smaller parts which resemble simpler, more familiar ideas

#8 Look for and express regularity in repeated reasoning (Generalize)

- Students are:
  - o Developing general methods, rules, or short cuts and determining when they are appropriate
- Teachers develop this skill by:
  - o Facilitating activities which allow for students' "aha!" moments and then helping them use it to develop "rules" based on repeated trials with a process

The following table outlines all content standards and learning objectives taught in the Algebra 2 and Algebra 2 Advanced courses. Standards designated with (+) are specific for the Algebra 2 Advanced course.

Number and Quantity		
The Complex Number System		
Standards	Learning Objectives	
	Algebra 2	Algebra 2 Advanced (additional topics)
N-CN.1 Perform arithmetic operations with complex numbers.  1. Know there is a complex number <i>i</i> such that <i>i</i> <sup>2</sup> = -1, and every complex number has the form a + b <i>i</i> where a and b real numbers.	<ul> <li>Understand the structure of number sets given that all real numbers are a subset of a larger group of Complex Numbers which includes imaginary numbers</li> <li>Define and use the imaginary unit i</li> </ul>	
N-CN.2 Perform arithmetic operations with complex numbers.  2. Use the relation i² = −1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	<ul> <li>Perform operations with complex numbers including addition, subtraction, multiplication, and division</li> <li>Use conjugates to simplify complex fractions</li> <li>Evaluate powers of i</li> </ul>	
N-CN.7 Use complex numbers in polynomial identities and equations using polynomials with real coefficients.  7. Solve quadratic equations using real coefficients that have complex solutions.	<ul> <li>Use the discriminant to determine if a quadratic will have complex solutions</li> <li>Find complex solutions and zeros of quadratic equations using square roots, completing the square, and the quadratic formula</li> </ul>	

N-CN.8 Use complex numbers in polynomial identities and equations using polynomials with real coefficients.  8. (+) Extend polynomial identities to the complex numbers. For example, rewrite x² + 4 as (x + 2i)(x - 2i).	<ul> <li>Use long/synthetic division to find all complex zeros of a polynomial given a known rational root</li> <li>Understand the Complex Conjugate Theorem to find conjugate pairs of complex zeros</li> </ul>	+ Write a polynomial equation as the product of complex linear factors
N-CN.9 Use complex numbers in polynomial identities and equations using polynomials with real coefficients.  9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.		<ul> <li>+ Use Descartes' Rule of Signs to determine the possible number and nature of a polynomial's zeros</li> <li>+ Use the Fundamental Theorem of Algebra to determine the number of complex solutions given the degree of a polynomial</li> </ul>
Algebra		
Seeing Structure in Expressions (A-SSE)		
A-SSE.1.a Interpret the structure of polynomial and rational expressions.	<ul> <li>Identifying and classifying polynomials</li> <li>Define: coefficient, term, constant term, degree, standard form of a polynomial, etc.</li> </ul>	
<ol> <li>Interpret expressions that represent a quantity in terms of its context.</li> <li>Interpret parts of an expression, such as terms, factors, and coefficients</li> </ol>		

A-SSE.1.b Interpret the structure of expressions.  1. Interpret expressions that represent a quantity in terms of its context. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1 + r) <sup>n</sup> as the product of P and a factor not depending on P.	<ul> <li>Use mathematical models to solve real-world problems</li> <li>Graph analysis and transformations</li> <li>Understand the effect of changing the value of constants within expressions</li> </ul>	
A-SSE.2 Interpret the structure of expressions.  2. Use the structure of an expression to identify ways to rewrite it.	<ul> <li>Apply the order of operations</li> <li>Solve equations for a specified variable</li> <li>Apply the algebraic properties of equality and inequalities</li> <li>Apply the properties of exponents</li> <li>Employ factoring techniques for polynomials</li> <li>Quadratics: convert standard to vertex form</li> <li>Convert repeating decimals into rational expressions</li> </ul>	
A-SSE.4 Write expressions in equivalent forms to solve problems.  4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. For example, calculating mortgage payments.	<ul> <li>Identify common ratios and differences</li> <li>Evaluate geometric and arithmetic series</li> <li>Use alternate formulas to calculate the sum of geometric and arithmetic series.</li> <li>Express and evaluate series using sigma notation</li> </ul>	

# Arithmetic with Polynomials and Rational Expressions (A-APR) A-APR.1 Add, subtract, and multiply polynomials Perform arithmetic operations on quadratic and Divide polynomials using long and synthetic higher order polynomials. division 1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; and add, subtract, and multiply polynomials. • Apply the remainder and factor theorems to A-APR.2 Understand the relationship between zeros and determine the zeros of a polynomial Analyze and use depressed polynomials to factors of polynomials. factor polynomials completely 2. Know and apply the Remainder Theorem: For a polynomial dividend p(x) and a number a, the remainder of the division by divisor (x - a) is p(a). Thus p(a) = 0 if and only if the divisor (x - a) is a factor of p(x). A-APR.3 Understand and apply the Fundamental Understand the relationship between zeros and theorem of Algebra factors of polynomials. Find roots and zeros of polynomials Interpret and then graph polynomial functions 3. Identify the zeros of polynomials when suitable Understand end behavior of polynomial factorizations are available, and use the zeros to functions construct a rough graph of the function defined by the polynomial.

A-APR.4 Use polynomial identities to solve problems.  4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.	<ul> <li>Solve polynomial equations</li> <li>Find rational roots</li> <li>Find complex roots (interpret discriminants)</li> <li>Use the complex conjugates theorem</li> <li>Understand and apply the rational root theorem to list and test possible zeros</li> </ul>	• + Use Descartes' rule of signs
A-APR.5 Use polynomial identities to solve problems.  5. (+) Know and apply the Binomial Theorem for the expansion of (x + y) <sup>n</sup> in powers of x and y for a positive integer n, where x and y are any numbers with coefficients determined for example by Pascal's triangle.	Use Pascal's triangle to expand binomials	<ul> <li>+ Use the binomial expansion theorem</li> <li>+ Relate Pascal's triangle to the binomial expansion theorem</li> </ul>
A-APR.6 Rewrite rational expressions including but not limited to linear and quadratic denominators.  6. Rewrite simple rational expressions in different forms. For example, write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.	<ul> <li>Simplify rational expressions</li> <li>Divide polynomials using long and synthetic division</li> <li>Use the remainder theorem to write polynomials as a product of the quotient and divisor, plus the remainder</li> <li>Add, subtract, multiply, and divide rational expressions</li> <li>Rationalize the denominator of rational expressions</li> </ul>	

A-APR.7 Rewrite rational expressions including but not limited to linear and quadratic denominators.  7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	+ State the excluded values for simplified rational expressions	<ul> <li>+ State the excluded values for simplified rational expressions, distinguishing between point and infinite discontinuity</li> </ul>
Creating Equations (A-CED)		
A-CED.1 Create equations, including but not limited to simple root functions, that describe numbers or relationships.  1. Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	<ul> <li>Write equations and inequalities of linear, quadratic, absolute value, rational, radical, and exponential functions in a single variable.</li> <li>Solve real-world problems in a single variable.</li> </ul>	
A-CED.2 Create equations, including but not limited to simple root functions, that describe numbers or relationships.  2. Create equations in two or more variables to represent relationships between quantities, and graph those equations on coordinate axes with labels and scales.	<ul> <li>Write equations and inequalities of linear, quadratic, absolute value, rational, radical, and exponential functions in two or more variables.</li> <li>Solve real-world problems in two or more variables.</li> </ul>	

A-CED.3 Create equations, including but not limited to simple root functions, that describe numbers or relationships.  3. Represent constraints by equations or inequalities, and by systems of equations or inequalities, and interpret the solutions as viable or nonviable options in a modeling context.	<ul> <li>Write and solve systems of equations in two and three variables</li> <li>Interpret solutions within context and apply units appropriately</li> </ul>	Linear Programming
A-CED.4 Create equations, including but not limited to simple root functions, that describe numbers or relationships.  4. Rearrange formulas to highlight a quantity of interest using the same reasoning for solving equations.	<ul> <li>Solve systems using the substitution method</li> <li>Find the inverse of a function</li> </ul>	
Reasoning with Equations and Inequalities (A-REI)		
A-REI.2 Understand and explain solving simple, radical, and rational equations as a process of reasoning.  2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	<ul> <li>Solve radical equations (including rational exponents) and inequalities</li> <li>Identify extraneous solutions</li> <li>Solve rational equations by cross multiplying and by using the least common denominator</li> </ul>	State the domain of radical equations

A-REI.3.1 Solve equations and inequalities in one variable.  3.1 Solve, graph, and interpret one-variable, absolute value equations and inequalities.	<ul> <li>Solve one-variable equations and inequalities involving absolute value, graphing the solutions and interpreting them in context.</li> </ul>	
A-REI.11 Represent and solve equations and inequalities graphically, combining polynomial, rational, radical, absolute value, and exponential functions.  11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find approximate solutions (e.g., using technology to graph the functions, make tables of values, or find successive approximations); include cases where functions are linear, polynomial, rational, absolute value, exponential, and logarithmic.	<ul> <li>Solve systems that combine linear, polynomial, rational, radical, absolute value, exponential, and logarithmic functions by creating tables and using technology to graph functions and find approximate solutions.</li> <li>Interpret the point(s) of intersection for two or more equations as the solution to the system</li> </ul>	Solve rational inequalities

## **Functions**

# **Interpreting Functions (F-IF)**

## F-IF.4

Interpret functions that arise in applications in terms of the context, emphasizing the selection of appropriate models.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

- Solve systems by graphic and algebraic methods.
- Analyze graphs of quadratics with focus and directrix
- Graph polynomial and rational functions
- Analyze the graphs of exponential functions, including using transformations
- Solve exponential equations
- Graph  $y = \sin x$ ,  $y = \cos x$ , and  $y = \tan x$ ; analyze key characteristics
- Apply transformations (amplitude, Vertical Shift, Periodic Functions, Period Change, Phase Shift) to trigonometric functions
- Find maximum and minimum values of quadratic functions
- Identify intervals for which the function is increasing/decreasing
- Graph quadratic and polynomial functions using x-intercepts
- Describe the end behavior of functions
- Write equations of parabolas
- Identify polynomial functions
- Identify even and odd functions
- Identify the periodicity of periodic functions

F-IF.5 Interpret functions that arise in applications in terms of the context emphasizing the selection of appropriate models.  5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	<ul> <li>Graph polynomial, rational and exponential and trigonometric functions and analyze key characteristics, including, but not limited to domain and range</li> </ul>	
F-IF.6 Interpret functions that arise in applications in terms of the context emphasizing the selection of appropriate models.  6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	<ul> <li>Graph linear, quadratic and exponential equations and determine the rate of change between specified points</li> <li>Use data to create a mathematical model (such as regression)</li> </ul>	
F-IF.7.b Analyze functions using different representations focusing on using key features to guide selection of appropriate model function types.  7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-	<ul> <li>Identify the key characteristics and graph radical, piecewise, step, and absolute value functions</li> </ul>	

defined functions, including step functions and absolute value functions.		
F-IF.7.C Analyze functions using different representations focusing on using key features to guide selection of appropriate model function types.	<ul> <li>Graph polynomial functions (including quadratic)</li> <li>Identify the key characteristic of graphs including end behavior and zeros</li> </ul>	
7. Graph functions expressed symbolically and show key features of the graph (by hand in simple cases and using technology for more complicated cases.) c. Graph polynomial functions, identifying the zeros when suitable factorizations are available, and show end behavior.		
F-IF.7.E Analyze functions using different representations focusing on using key features to guide selection of appropriate model function types.  7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	<ul> <li>Graph exponential growth and decay functions</li> <li>Graph natural base functions</li> <li>Graph logarithmic functions</li> <li>Graph and explore characteristics of the six primary trigonometric functions.</li> </ul>	<ul> <li>Graph inverse trigonometric functions</li> <li>Include phase shift for transformations on trigonometric functions</li> </ul>

F-IF.8 Analyze functions using different representations focusing on using key features to guide selection of appropriate model function types.  8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function	<ul> <li>Rewrite functions into different but equivalent forms</li> <li>Identify the key characteristics of the graph by analyzing the equation</li> </ul>	Given a sinusoidal graph, write the equation in multiple forms, including sine or cosine with negative coefficient or phase shift
F-IF.9 Analyze functions using different representations focusing on using key features to guide selection of appropriate model function types.  9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	Compare the key features of two functions that are in different forms	
Building Functions (F-BF)		
F-BF.1.B Build a function that models the relationship between two quantities or functions.  1. Write a function that describes a relationship between two quantities. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.	Write a function which models a relationship between two variables (e.g. model for projectile motion)	

# F-BF.3

Build new functions from existing functions, including but not limited to simple, radical, rational, and exponential functions, and emphasize the common effect of each transformation across function types.

3. Identify the effect on the graph of replacing f(x)by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

- Identify transformations of a function from a graph
- Define the effect of a transformation on the graph of a function including amplitude and vertical shifts.
- Explore the effect of constants on the graph of an equation through the use of technology.
- Prove algebraically that a function is even or odd.
- Graphically identify if a function is even or odd.

- Define the effect of a transformation on the graph of a function including amplitude, vertical shifts, and phase shifts
- Algebraically and graphically identify sine and cosine as even and odd functions

#### F-BF.4

Build new functions from existing functions, including but not limited to simple, radical, rational, and exponential functions, and emphasize the common effect of each transformation across function types.

- Find inverse functions.
- a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example,  $f(x) = 2x^3$  or f(x) = (x + 1)/(x - 1) for  $x \ne 1$ .

- Find the inverse of a function.
- Verify the inverses of nonlinear functions.
- Use inverse properties of logarithmic and exponential functions to solve equations
- Define and evaluate inverse trigonometric functions based on restricted domains
- Use inverse trigonometric functions to solve trigonometric equations

Linear, Quadratic, and Exponential Models (F-LE)		
F-LE.4 Construct and compare linear, quadratic, and exponential models and solve problems.  4. For exponential models, logarithmically express the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e. In addition, use technology to evaluate logarithms as solutions for exponentials.	<ul> <li>Convert between exponential and logarithmic form</li> <li>Evaluate exponential and logarithmic expressions using technology</li> </ul>	Growth and decay applications problems
<b>F-LE.4.1</b> 4.1 Prove simple laws of logarithms.	Prove simple laws of logarithms	
<b>F-LE.4.2</b> 4.2 Use the definition of logarithms to translate between logarithms in any base.	<ul> <li>Apply the change of base formula to evaluate logarithmic expressions</li> </ul>	
<b>F-LE.4.3</b> 4.3 Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.	<ul> <li>Understand the relationship between the laws of exponents and the laws of logarithms</li> <li>Expand, condense, and evaluate logarithms using and applying the laws of logarithms and exponents</li> </ul>	
Trigonometric Functions (T-TF)		
F-TF.1 Extend the domain of trigonometric functions using the unit circle.  1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the	<ul> <li>Define and draw angles in standard position</li> <li>Define radian measure</li> <li>Convert between degree and radian measure</li> </ul>	

angle.		
F-TF.2 Extend the domain of trigonometric functions using the unit circle.  2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	<ul> <li>Understand that coterminal angles define the domain of trigonometric function as (-∞,∞)</li> <li>Define positive, negative, clockwise, and counterclockwise angles.</li> <li>Identify and use reference angles to evaluate trigonometric functions</li> <li>Utilize special right triangles in the unit circle to evaluate trigonometric functions of any angle</li> </ul>	
F-TF.2.1 2.1 Graph the six primary trigonometric functions.	<ul> <li>Identify the characteristics of parent functions y = sinx, y = cosx and y = tanx</li> <li>Use transformations to change the period and amplitude</li> <li>Use transformations to translate parent functions</li> <li>Use parent functions and transformations to graph y = secx, y = cscx and y = cotx</li> </ul>	<ul> <li>Use transformations to change the period and amplitude and phase shift</li> <li>Understand the relationship between phase shift and y = sinx and y = cosx</li> </ul>
F-TF.5 Model periodic phenomena with trigonometric functions.  5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	<ul> <li>Understand the role of a,b and d in the equation y = asin(bx) + d to determine the frequency, amplitude, vertical shifts of sinusoidal models</li> </ul>	<ul> <li>Include phase shift in the understanding of a,b, c and d in the graph of y = asin b(x+ c) + d</li> <li>Write and use trigonometric equations for sinusoidal modeling problems</li> </ul>
F-TF.8 Prove and apply trigonometric identities.  8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ given	<ul> <li>Use x = cos(θ) and y = sin(θ) from the unit circle to derive the Pythagorean identity</li> <li>Use trigonometric identities to evaluate trigonometric functions, simplify trigonometric expressions, and verify other trigonometric</li> </ul>	<ul> <li>Use the sum and difference, and double and half-angle identities to evaluate trigonometric functions, simplify trigonometric expressions, and verify other trigonometric</li> </ul>

$sin(\theta)$ , $cos(\theta)$ , or $tan(\theta)$ and the quadrant of the angle.	identities	identities  • Apply trigonometric identities to solve trigonometric equations		
Geometry  Expressing Geometric Properties with Equations (G-GPE)				
Statistics and Probability  Interpreting Categorical and Quantitative Data (S-ID	)			
S-ID.4 Summarize, represent, and interpret data on a single count or measurement variable.  4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are	<ul> <li>Calculate probabilities using normal distributions</li> <li>Use z-scores and the standard normal table to find probabilities</li> <li>Recognize normal data sets</li> </ul>			

data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.				
Making Inferences and Justifying Conclusions (S-IC)				
S-IC.1 Understand and evaluate random processes underlying statistical experiments.  1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	<ul> <li>Distinguish between populations and samples</li> <li>Analyze hypotheses</li> </ul>			
S-IC.2 Understand and evaluate random processes underlying statistical experiments.  2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, if a model says a spinning coin falls heads up with probability 0.5, would a result of five tails in a row cause one to question the model?	<ul> <li>Analyze types of sampling methods</li> <li>Resample data using a simulation to analyze a hypothesis</li> </ul>			
S-IC.3 Make inferences and justify conclusions from sample surveys, experiments, and observational studies.  3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization	<ul> <li>Analyze methods of collecting data</li> <li>Recognize bias in survey questions and sampling</li> <li>Recognize how randomization applies to experiments and observational studies</li> </ul>			

relates to each.		
S-IC.4 Make inferences and justify conclusions from sample surveys, experiments, and observational studies.  4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	<ul> <li>Find margins of error for surveys</li> <li>Estimate and analyze population parameters</li> </ul>	
S-IC.5 Make inferences and justify conclusions from sample surveys, experiments, and observational studies.  5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	<ul> <li>Organize data from an experiment with two samples</li> <li>Make inferences about a treatment</li> </ul>	
S-IC.6 Make inferences and justify conclusions from sample surveys, experiments, and observational studies.  6. Evaluate reports based on data	Evaluate reports based on data	

Using Probability to Make Decisions (S-MD)			
S-MD.6+ Use probability to evaluate simple and complex outcomes of decisions.  6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).		<ul> <li>Prove whether events are independent or dependent using the sample space</li> <li>Calculate probabilities of independent and dependent events</li> <li>Calculate conditional probabilities</li> <li>Find probabilities of compound events</li> <li>Use more than one probability rule to solve real-life problems</li> </ul>	
S-MD.7+ Use probability to evaluate simple and complex outcomes of decisions.  7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).		<ul> <li>Use more than one probability rule to solve real-life problems</li> <li>Construct and interpret probability and binomial distributions</li> </ul>	